Math 10B with Professor Stankova
Worksheet, Discussion #5; Thursday, 2/7/2019
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Distinguishable Boxes

Concepts

1. A lot of the cases with distinguishable boxes or dogs have already been covered in this class via our old methods of permutations and combinations. The new and more pertinent case that hasn't been covered is indistinguishable balls and distinguishable boxes. For this, if we have b identical biscuits and d distinguishable dogs, there are $\binom{b+d-1}{b}$ ways to distribute the biscuits to the dogs.

Examples

2. Suppose I am catering from Yali's and want to buy sandwiches to feed 60 students. How many ways can I do this if they have 8 sandwich options? How many ways can I do this if I want to get at least 2 of each sandwich?

3. How many ways can I distribute 30 course spots amongst 4 grades (freshmen, sophomores, juniors, seniors) so that there are no more than 10 freshmen in the course?

Problems

4. True False In first example, since each student is getting a sandwich, the balls are sandwiches and the urns are students.

5. True False The number of ways to distribute 8 identical balls to 12 urns is the same as the number of ways to distribute 11 identical biscuits to 9 dogs.

6. How many ways are there to deal hands of five cards to each of six players from a deck containing 52 different cards?

7. How many ways can you deal the 52 cards of a deck to 4 people so that everyone gets 13 cards and the oldest player gets the queen of spades?

8. You are buying bread for a soccer team. A bread shop has white bread, whole wheat, rye, sesame seed, pumpernickel, and gluten free. How many ways are there to choose 2 dozen loaves of bread?

9. How many solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ if all are positive integers and $x_3 \leq 3$?

10. How many 3 digit numbers have a sum of digits equal to 9?

11. How many 7 digit decreasing numbers are there? One example is 9777650.

12. (Challenge) How many numbers less than $1,000,000$ have the sum of their digits equal to 10?

Indistinguishable Boxes

Concepts

13. The value $S(n,k)$ represents the number of ways we can distribute n distinguishable biscuits to k identical dogs or identical non-empty piles. The value $p_k(n)$ represents the number of ways to partition n identical biscuits to k identical non-empty piles.

    There is a recursive formula for the Stirling numbers (but not partition numbers) as $S(n+1,k) = kS(n,k) + S(n,k-1)$.

Example

14. How many ways are there to split 28 distinct students up into at most 6 different groups if the groups are not numbered? What if the students are not distinct?

Problems

15. True False The only way to determine what $S(5,3)$ is to list out all the possibilities.

16. True False The only way to determine what $p_3(5)$ is to list out all the possibilities.

17. True False In order to determine the number of ways to distribute 10 distinguishable items into 3 identical boxes so that each box has at least 2 items, we can place one item in each box and this problem reduces to the regular case of distributing 7 items in 3 identical boxes which is $S(7,3)$ ways.

18. True False In order to determine the number of ways to distribute 10 identical items into 3 identical boxes so that each box has at least 2 items, we can place one item in each box and this problem reduces to the regular case of distributing 7 items in 3 identical boxes which is $p_3(7)$ ways.

19. There are 14 students that want to break off into 3 non-empty study groups. How many ways can this occur?

20. How many ways can 5 different employees be assigned to 3 identical offices when each office must have at least one person?

21. I want to store my 200 Yu-gi-oh cards in 4 different identical boxes. How many ways can I do this if some boxes are allowed to be empty?

22. How many ways are there to split 15 identical marbles to 5 different non-empty groups?

23. How many ways are there of distributing 30 identical objects into 3 boxes if each box must have at least 5 items?